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The Relationship between Seats and Votes in Multiparty Systems

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The relationship between a party's popular vote share and legislative seat share—its seats–votes *swing ratio*—is a key characteristic of democratic representation. This article introduces a general approach to estimating party-specific swing ratios in multiparty legislative elections, given results from only a single election. I estimate the joint density of party vote shares across districts using a finite mixture model for compositional data and then computationally evaluate this distribution to produce parties' expected change in legislative seats for plausible changes in their vote share. The method easily extends to systems with any number of parties, employing both majoritarian and proportional electoral rules. Applications to legislative elections in the United States, United Kingdom, Canada, and Botswana demonstrate how parties' swing ratios vary both within countries and over time, indicating that parties under majoritarian electoral rules are subject to unique and possibly divergent geographic–political constraints.

1 Introduction

In democratic legislative elections, the set of rules and procedures for converting voters' choices into political parties' seat shares is referred to as the *electoral system*. There are a wide variety of electoral systems in use around the world, each employing a different set of voting rules, electoral formulas, district magnitudes, and so forth (Lijphart 1999; Farrell 2001; Colomer 2004; Gallagher and Mitchell 2005; Golder 2005). For researchers interested in the *consequences* of electoral system design, one of the most important characteristics of electoral institutions is how they affect the sensitivity of parties' legislative seat shares to *changes* in their popular vote shares—a fundamental component of democratic representation and electoral accountability. In proportional or mixed systems, vote shares and seat shares covary in a roughly one-to-one manner. But in majoritarian systems, small vote swings can produce much larger changes in parties' shares of legislative seats. In the language of electoral systems research, majoritarian institutions sacrifice a certain amount of *proportionality* for a larger degree of *responsiveness* (Powell 2000).

By influencing how exposed political parties are to changing voter preferences, electoral systems shape both the incentives and the constraints that politicians face when strategizing how to win elections, gain legislative power, and implement policy. As a result, parties and politicians operating under different electoral systems are expected to act and govern in systematically different ways, from country to country and election to election (Bawn and Thies 2003; Rodden 2010). Variation in electoral institutions has been linked to outcomes as diverse as rates of legislative turnover (Matland and Studlar 2004), pork barrel politics (Lancaster 1986), welfare expenditures (Crepaz 1998; Jusko 2009), price levels (Rogowski and Kayser 2002; Linzer and Rogowski 2008), trade protection (Park and Jensen 2007), economic redistribution (Iversen and Soskice 2006), human rights protection (Cingranelli and Filippov 2010), and even levels of government corruption (Kunicová and Rose-Ackerman 2005; Chang and Golden 2006).

Empirical testing of theories such as these raises difficult measurement issues. The mechanism behind most electoral system effects is hypothesized to operate at the level of the individual party. Yet, no fully

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general, party-level measure of electoral institutional pressures currently exists. Thus, researchers frequently use *national*-level measures of the ''majoritarian-ness'' of an electoral system, instead—features such as electoral system type or average district magnitude. The implied assumption is that the effects of electoral systems apply equally to every party, regardless of their size or distribution of support across districts. This assumption may be plausible in countries using proportional electoral rules or even in majoritarian countries with only two parties (as there, parties tend to be of roughly equivalent size and votes lost by one party are necessarily won by the other), but it is not true in general. In most democracies, many more than two parties contest and win seats in national legislative elections. Even among countries employing single-member legislative districts, fewer than one-third are true two-party systems (Reynolds, Reilly, and Ellis 2008). To the extent that parties in multiparty majoritarian systems experience divergent electoral constraints, it may be necessary to refine or reconsider a variety of findings that have been previously attributed to differences in electoral systems.

In this article, I develop a statistical procedure for estimating the relationship between political parties' legislative seats and votes in multiparty systems using district-level voting data from only a single election. In particular, I show how to estimate parties' *swing ratio* or the ''percentage change in legislative seats associated with a 1% change in legislative votes'' (Niemi and Fett 1986, 76). The swing ratio is a measure of the sensitivity of *individual parties'* seat shares to changes in voters' preferences for representation. It is analogous to, but distinct from, *system-level* measures of electoral responsiveness. Within countries, as well as over time, parties' swing ratios vary, depending upon a country's electoral rules, the extent of malapportionment (districts containing unequal numbers of voters), the number of parties, parties' levels of popular support, and the geographic distribution of that support (Gudgin and Taylor 1979; Taagepera and Shugart 1989). Large swing ratios—typically three or more—indicate parties that are highly vulnerable to defeat at the polls but that also stand to gain legislative influence rapidly if their vote shares increase. Seat shares of parties with smaller swing ratios will be less affected by changes in their vote shares. Small regional parties, for example, will not typically reap the same gain in legislative seats for a 1% increase in vote share as would large national parties in the same system.

The technique is broadly applicable to many different forms of multiparty electoral competition. It estimates the joint distribution of parties' level of electoral support across districts, based on a finite mixture model for compositional data. It then evaluates this density estimate using Monte Carlo simulation, to determine how changes in each party's vote and seat shares translate into losses or gains for all other competing parties, at both the district and the national levels. The method easily extends to elections with multiple patterns of partially contested or uncontested districts, generalizes to systems with any number of political parties (including two), and allows for the composition of the party system to change over time.[1] This not only permits estimation of how different parties are subject to greater or lesser majoritarian pressures, but it also makes it possible to capture election-by-election dynamics in parties' swing ratios. The simulation results can be further extended to produce parties' local seats–votes curves around each election outcome. The estimation technique is implemented as a self-contained and easy-to-use software package for R (R Development Core Team 2011; Linzer 2012).

There is a lengthy methodological literature on the estimation of parties' swing ratios and other summary measures of electoral systems, to which this article aims to contribute. Current statistical models of the seats–votes relationship in multiparty systems (e.g., Taagepera 1986; King 1990; Calvo 2009) are designed to estimate the overall responsiveness of an electoral system, rather than separate swing ratios for each individual party. A range of methods have been proposed for estimating party swing ratios in two-party systems (e.g., Gelman and King 1990b, 1994; King and Gelman 1991; Jackman 1994), but these produce faulty inferences when applied to multiparty data. They require a researcher to either disregard the vote shares received by all but two parties or aggregate multiple parties' vote shares into just two groups, which unrealistically assumes that voters never switch preferences among parties within each bloc. The two-party methods also constrain both parties in the analysis to have equivalent swing ratios, which is almost never the case in multiparty systems.

[1]The approach can be applied to proportional as well as majoritarian systems but is of primary interest for parties in majoritarian systems, as in proportional systems it is expected that parties' swing ratios will approximate one.

Knowledge of parties' individual swing ratios can create new opportunities for comparative electoral systems research. I demonstrate the technique with application to historical election data in Canada, the United States, the United Kingdom, and Botswana—all democracies employing majoritarian electoral rules, but with party systems varying from two-party to multiparty to, in the case of Botswana, single-party dominant. These countries represent the three main forms of majoritarian party competition around the world.

2 The Seats–Votes Relationship

The swing ratio describes the expected change in a party's share of legislative seats following hypothetical changes in its share of the national vote. One approach to assessing this quantity might compare election outcomes across multiple years, to see how parties' seat and vote shares actually covaried (e.g., Kendall and Stuart 1950; Tufte 1973; King and Browning 1987; King 1990). However, as discussed by Niemi and Fett (1986) and Jackman (1994), there are weaknesses to multiyear methods. They require data from a large number of elections to achieve stable estimates. They are unable to capture over-time changes in parties' seats–votes relationship. Nor do they effectively accommodate changes to a country's party system or handle uncontested electoral districts in a consistent manner.

Models for estimating the swing ratio based upon data from a *single* election represent a superior approach. For the case of two-party competition, Gelman and King (1990b, 1994), King and Gelman (1991), and Jackman (1994) have proposed methods that extrapolate parties' seat allocations under alternative election outcomes by combining estimates of how parties' vote shares are distributed across districts with information about the election-to-election variability in districts' vote outcomes. In two-party systems, a single seats–votes curve is sufficient to estimate the swing ratio for both parties. But in multiparty systems, swing ratio estimation requires a separate seats–votes curve for each party.

There are four distinctive features of legislative elections in multiparty systems that must be considered when modeling the seats–votes relationship:

1. The number of parties in different countries can vary from as few as three to over a dozen. Even within countries, the roster of parties contesting elections in multiparty systems can be highly variable from year to year. In two-party systems, the same parties tend to compete repeatedly over long stretches of time.
2. In almost all multiparty elections, not every party fields a candidate in every district. This produces multiple patterns of party contestation—different configurations of parties that compete against one another at the district level. Uncontested districts in which only one party competes are rare. In two-party contests, both parties tend to compete in most districts, with a non-negligible quantity remaining uncontested.
3. Malapportionment can be quite large in multiparty systems, although this is also true of many two-party systems (Grofman, Koetzle, and Brunell 1997; Samuels and Snyder 2001). Consequently, in many multiparty elections, parties' average district vote shares can differ significantly from their total (national) vote shares.
4. From a practical standpoint, in many countries, data describing features of electoral districts other than the outcome of the vote—for example, measures of economic or social characteristics commonly associated with election outcomes—may be scarce or nonexistent.

In sum, not only can two-party methods not be directly applied to multiparty data, but also they are not even easily extensible to the multiparty setting. The techniques offered by Gelman and King require analyzing multiple years of election results in a preliminary stage to estimate the fundamental variability in parties' district-level support. This is not possible if the same parties do not contest the same districts in repeated years. Nor is it feasible to incorporate district-level covariates, as in Gelman and King (1994), if the data are unavailable. It is also not evident from a theoretical standpoint that, for example, a variable measuring incumbency—which assumes a prominent position in the model of Gelman and King—will have a consistent meaning in multiparty systems, where the occurrence of coalition governments implies that more than one party can be considered to control the government (Laver and Schofield 1998). And, although uncontested districts are not generally present in multiparty elections, malapportionment can be considerable. Because district size tends to correlate with the vote shares received by certain parties in multiparty elections, it is important to model malapportionment directly, rather than to make the simplifying assumption that all districts are of equal size (Jackman 1994).

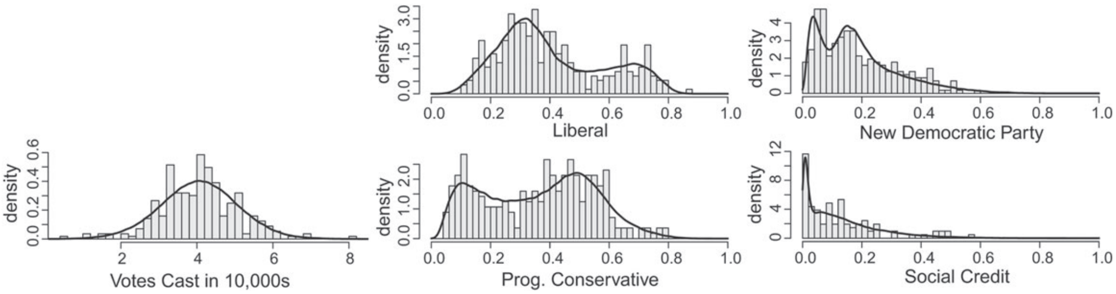

Fig. 1 Marginal distributions of parties' vote shares (in districts where they fielded a candidate) in the 1979 Canadian federal election. The leftmost histogram shows the total number of votes cast in each electoral district, in units of 10,000. Overlaid curves represent the marginal distributions of the joint density estimate $F(Y; \widehat{\boldsymbol{\pi}}, \widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\Sigma}})$ described below.

3 A Multiparty Seats–Votes Model

I outline a two-step method for estimating the swing ratios of individual parties in multiparty systems. Building upon the approach of Gelman and King (e.g., Gelman and King 1990b, 1994; King and Gelman 1991), I first estimate a density function that captures the underlying joint distribution of party vote shares and number of votes cast across districts. I then employ Monte Carlo simulation methods to generate alternative, hypothetical election outcomes that are consistent with the density estimate. Repeated computational simulation provides the basis for estimating each party's swing ratio. I also show how the simulation results may extend to estimate the interrelationships among parties' potential national vote and seat shares, as well as parties' local seats–votes curves and other quantities of interest.

As a running illustration, I will examine the Canadian federal election of 1979, in which four parties—Progressive Conservative, Liberal, New Democratic, and Social Credit—won seats in Parliament and combined to receive over 98% of the popular vote. The election was held under single-member district plurality rule. The outcome of this election makes it an interesting test case: although the Liberal Party won the popular vote with 40% and received a seemingly fair 40% of the seats in the House of Commons, the Progressive Conservative Party ended up with a *larger* share of the legislative seats, 48%, despite winning a smaller share of the votes, 36%. Support for these parties was closely tied to geography, with Liberal voters highly concentrated in populous Quebec; Progressive Conservatives doing well in the western provinces; and Social Credit winning six seats in Quebec and zero in the rest of the country.[2] The joint and marginal distributions of party vote shares across districts demonstrate both asymmetry and multimodality (Fig. 1).

3.1 *Multiple Patterns of Party Contestation*

It is common in multiparty legislative elections for small or regional parties to only nominate candidates in a subset of electoral districts. When this happens, we observe a number of distinct *patterns of party contestation*: particular combinations of parties that compete against one another in at least one district. As the number of different patterns of party contestation in multiparty legislative elections is rarely very large, I follow the idea of Tomz, Tucker, and Wittenberg (2002, 69) to perform "a separate analysis for each pattern of contestation" in producing the overall density estimate.

Let M represent the observed number of different patterns of party contestation, with N_m the number of districts in the mth pattern of party contestation. Denote as D the total number of legislative districts in a country, so $D = \sum_{m=1}^{M} N_m$, and N_m/D is the share of the districts in the mth group. Also denote as P_m the total number of parties fielding a candidate in the mth pattern of party contestation. In the 1979 Canadian federal election, there were $M = 2$ patterns of party contestation.[3] The Progressive Conservative, Liberal, and New Democratic parties all fielded candidates in each of the $D = 282$ election districts. In addition, the Social Credit Party competed in 103 of those districts. Thus, $N_1 = 179$ and $P_1 = 3$ for the three-party districts and $N_2 = 103$ and $P_2 = 4$ for the four-party districts.

[2] The source of the Canadian election data is Brancati (2007).

[3] To be precise, that is, among the four parties ultimately winning seats. Including the remaining small parties does not substantively alter the model estimates.

3.2 *Estimating the Distribution of Vote Shares*

Party vote shares are compositional variables that sum to 100% within each district.[4] To remove this constraint, and with no loss of information, I transform the P_m party vote shares in each district into $P_m - 1$ variables formed by the logarithm of the ratio of each party's vote share to the vote share received by a static *reference* party. The resulting log-ratios are continuous, unbounded, and not deterministically related. This transformation is a common approach to modeling compositional variables that facilitates estimation of statistical models using conventional distributional assumptions (Aitchison 2003). The choice of reference party is arbitrary and does not affect the results in any way.

Expressing as v_{ip} the share of the vote received by the pth party in the ith district ($p = 1 \ldots P_m$; $i = 1 \ldots N_m$), and making party one the reference party, the $P_m - 1$ log vote ratios are $\lambda_{i2} = \ln(v_{i2}/v_{i1}),\ \ \lambda_{i3} = \ln(v_{i3}/v_{i1}), \ldots, \lambda_{iP_m} = \ln(v_{iP_m}/v_{i1})$. Also denote as z_i the size or number of votes cast in the ith district. Putting these together, $\boldsymbol{Y}_m = (z, \lambda_2, \lambda_3, \ldots, \lambda_{P_m})$ is the $N_m \times P_m$ matrix of election results for the mth pattern of party contestation. Row vector Y_i is a single district. Treating z as a random variable enables the model to account for the effects of malapportionment.

In districts where one party runs unopposed, $P_m = 1$ for that pattern of party contestation, so no log-ratios can be calculated. Nor can it be assumed that the observed number of votes cast for a lone competing party, z_i, is equal to the number that would have been cast had the district been contested. In many cases, z_i is not even observed in uncontested districts, as unopposed candidates are either awarded the district's seat without any voting taking place or the number of votes cast goes unrecorded. Previous approaches have recognized that uncontested districts arise for systematic reasons and accordingly sought to adjust parties' vote shares in these districts to reflect levels of voter support for each party that are not exactly zero or 100% (e.g., Gelman and King 1994). Since my method does not require the calculation of an average district vote, rather than attempting to reconstruct estimates of district characteristics as if an uncontested district had been contested, I simply tally the number of uncontested districts N_m for each party and hold those values aside.

For the general case in which $P_m > 1$, I estimate the joint distribution of variables in $\boldsymbol{Y}_m$ using a finite mixture of multivariate normal densities.[5] One mixture model is estimated for each pattern of multiparty contestation. This model provides sufficient flexibility to fit a wide variety of patterns of legislative election results. No district-level covariates or any other data describing features of the legislative districts are necessary. This makes the method feasible to implement given the constraints of limited data availability, without sacrificing the quality of the fit of the model to the election data.

Index the component distributions of the mth mixture as $r = 1 \ldots R_m$. The parameters estimated for the model are the means $\boldsymbol{\mu}_r^m$ and covariance matrices $\boldsymbol{\Sigma}_r^m$ of each of the component multivariate normals, as well as a vector of mixing proportions π_r^m, which sum to one and represent the weights assigned to each component. The superscript m again signifies that separate mixture densities are estimated for each pattern of party contestation. Denote as $f_{MVN}(Y_i; \boldsymbol{\mu}_r^m, \boldsymbol{\Sigma}_r^m)$ the density function of the rth multivariate normal distribution evaluated at district i. The mixture model log-likelihood is

$$\ln L = \sum_{i=1}^{N_m} \ln \sum_{r=1}^{R_m} \pi_r^m f_{MVN}(Y_i; \boldsymbol{\mu}_r^m, \boldsymbol{\Sigma}_r^m). \tag{1}$$

This function can be maximized using the expectation–maximization (EM) algorithm (Dempster, Laird, and Rubin 1977; McLachlan and Krishnan 1997).[6] Once the model has been estimated, the category labels r hold no unique meaning. Since the objective in fitting the mixture model is purely for density

[4]If minor parties receive a residual share of the vote in certain districts, the vote shares of the major parties may be easily rescaled to sum to 100% by dividing each party's observed vote share by the total share of the vote received by all major parties in the district.

[5]The number of votes cast per district, z_i, has a lower (zero) bound that could be removed using a log transformation. In practice, however, values of this variable are so large that the mixture approximation places essentially no density at or below zero.

[6]Despite potential concerns about the robustness of likelihood-based estimation of finite mixture models (e.g., MacKay 2003, chap. 22), district-level election data tend to follow systematic patterns that are conducive to stable density estimates. In addition, only small numbers of component distributions are typically required, which helps avoid overfitting. An alternative Bayesian approach to estimating mixture models is employed by Gelman and King (1990b), King and Gelman (1991), and Jackman (1994). Either procedure is acceptable, as long as proper steps are taken to verify the fit of the model to the data. For the EM algorithm, this may require reestimating the model a small number of times with different starting values to ensure that the global maximum of the log-likelihood function has been found.

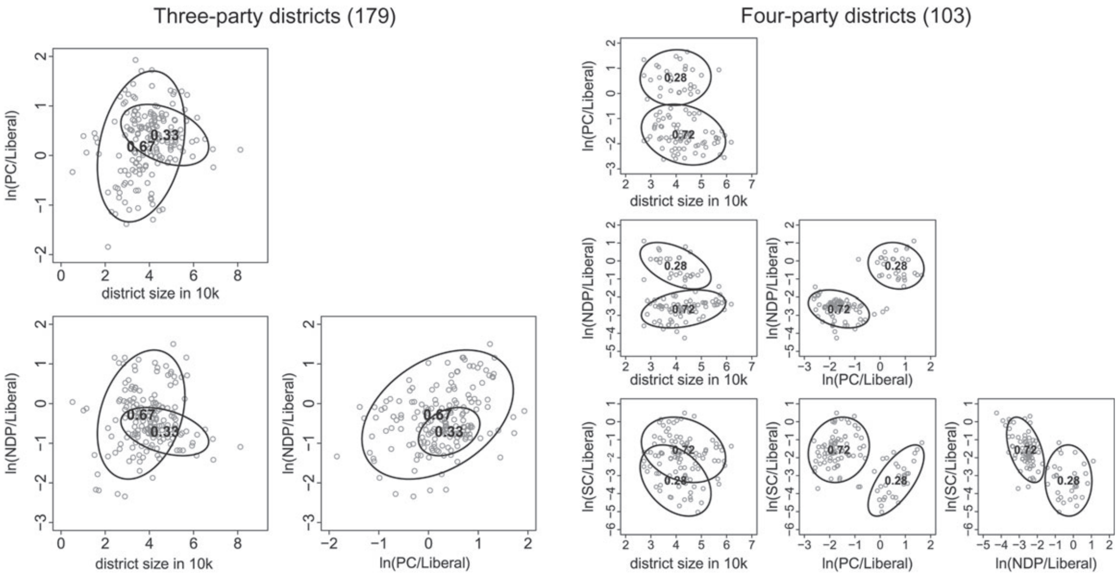

Fig. 2 Scatterplot matrices of the log-ratio vote outcomes (with respect to the Liberal Party) and district sizes. Points denote districts. At left are the 179 districts contested only by the Progressive Conservative, Liberal, and New Democratic Parties. At right are the 103 districts additionally contested by the Social Credit Party. The ovals represent the fitted mixture of multivariate normal distributions, drawn at two standard deviations of each component. The numbers at the center of each oval give the weights, $\widehat{\pi}_r$, of that component.

estimation, the individual components serve only to indicate the nature of variation in the observed district-level election outcomes.

Achieving an optimal fit of the mixture model within each pattern of party contestation requires careful selection of the number of component distributions. The goal is to select the smallest R_m that sufficiently captures the observed variation in the data. As even the largest legislatures will have at most a few hundred districts in a single pattern of party contestation, two or three component distributions are almost always sufficient. The preferred approach is to begin with $R_m = 1$, at which point a variety of visual diagnostics are available to evaluate model fit, as seen in Figs. 1, 2, and 3. In the event that the model does not satisfactorily detect the skewness or multimodality of the distribution of party vote shares, R_m may then be increased by increments of one, until a suitable fit is achieved.

The fitted mixture densities are then combined across patterns of party contestation, in proportion to the share of districts observed in each group. The resultant density function is an estimate of the underlying joint distribution of vote shares and district sizes across the *entire set* of contested electoral districts and is given by the weighted sum

$$F\big(Y; \widehat{\boldsymbol{\pi}}, \widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\Sigma}}\big) = \sum_{m=1}^{M^{c}} \frac{N_m}{D^{c}} \sum_{r=1}^{R_m} \widehat{\pi}_r^m f_{MVN}\big(Y; \widehat{\boldsymbol{\mu}}_r^m, \widehat{\boldsymbol{\Sigma}}_r^m\big). \tag{2}$$

Because uncontested districts have been set aside, I distinguish the total number of patterns of party contestation, M, from the number of patterns of party contestation in which two or more parties compete, M^{c}. Similarly, D^{c} denotes the number of districts excluding those that are uncontested. Each of the M^{c} mixture distributions only have positive density over their respective patterns of party contestation.

For the 1979 Canadian election, I fit a two-component model to the 179 districts where only the three national parties fielded candidates ($R_1 = 2$) and another two-component model to the 103 districts where a candidate from the Social Credit Party also competed ($R_2 = 2$). In both patterns of party contestation, the Liberal Party is used as the reference party for calculating the log vote ratios. The results are shown in Fig. 2. At right, the scatterplot matrix of election results in districts contested by all four parties reveals a bimodal clustering pattern that validates the use of mixture distributions. A relationship between district size and election outcome is also apparent in both patterns of contestation, which demonstrates the need to account for malapportionment. The overall, country-wide distribution of vote shares and district sizes is

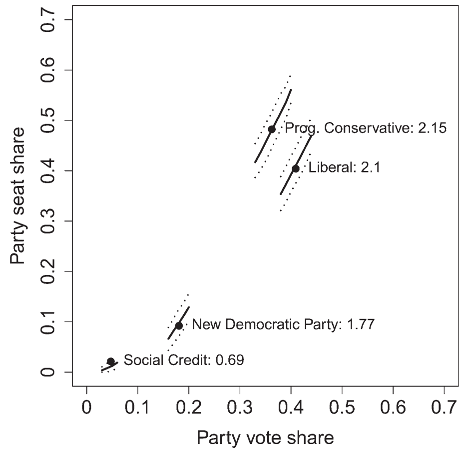

Fig. 3 Local seats–votes curves and 95% confidence intervals for parties contesting the 1979 Canadian federal election. Results are based on 50,000 simulations using the mixture densities in Fig. 2. Black dots denote the actual election result. Numbers following each party name represent that party's swing ratio.

the weighted sum of 179/282 times the three-party mixture plus 103/282 times the four-party mixture. The marginal densities of the estimated $F(Y; \widehat{\boldsymbol{\pi}}, \widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\Sigma}})$ are overlaid on the histograms in Fig. 1 to verify that the model has succeeded in capturing the underlying distribution of vote shares and votes cast.

3.3 *Evaluating the Swing Ratio Using Monte Carlo Simulation*

The density estimate $F(Y; \widehat{\boldsymbol{\pi}}, \widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\Sigma}})$ represents the plausibility of various district-level election outcomes—both vote shares and number of votes cast per district—given the conditions of that election. This distribution also indicates how, at the district level, parties are expected to jointly trade off voter support. In Fig. 2, for example, there is a positive correlation in the three-party districts between the log-ratios of New Democratic to Liberal Party vote shares and Progressive Conservative to Liberal Party vote shares. This indicates that as the Progressive Conservative Party increased its vote share relative to the Liberal Party, the New Democratic Party (NDP) did so as well. In the four-party districts, by contrast, there are two distinct types of election outcomes. The smaller group, labeled 0.28, represents districts where both the Progressive Conservative and New Democratic Party candidates ran strongly against the Liberal candidate but where the Liberal candidate was strongly preferred to the Social Credit Party candidate. In the larger cluster, an entirely different dynamic appears to have been at play, with Liberal candidates running very strongly against all others.

Aggregating up from the district level, the estimated density function can be used to assess covariation in parties' *national-level* vote shares and how this would carry through to their legislative seat shares. The variety of district-level vote changes that might increase or decrease a party's national-level vote share by 1% is theoretically infinite, and each of these will imply different changes in parties' national-level seat shares. But what the high- and low-density areas in Fig. 2 show—and what $F(Y; \widehat{\boldsymbol{\pi}}, \widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\Sigma}})$ captures—is that not every combination of district-level election outcomes is equally likely to occur. It is not just that the Social Credit Party would never have won 80% (or more) of the vote in a district. The density estimate indicates which *combinations* of party vote shares are more or less plausible at the district level and how parties' seat shares would vary as a result. Evaluating these relationships leads directly to the primary quantity of interest: the swing ratio, or expected change in a party's share of legislative seats, given a hypothetical 1% deviation in its national-level vote share from the observed election result.

The starting point for each party p is its proportion of national-level votes *actually won*,

$$\bar{V}_p = \frac{\sum_{i=1}^{D^c} z_i v_{ip}}{\sum_{i=1}^{D^c} z_i}. \tag{3}$$

The sum is taken only over the contested districts because z_i is recorded inconsistently (and may be unobserved) in uncontested districts. The observed proportion of legislative *seats* won by each party is $\bar{S}_p$, which, under a plurality (first-past-the-post) electoral rule, equals the number of districts in which party p wins the most votes, divided by D. Although $\bar{V}_p$ is calculated excluding votes cast in uncontested districts (even if that number is observed), $\bar{S}_p$ includes seats won in uncontested districts.

Let $\Pr(\tilde{S}_p|V_p)$ denote the distribution of potential seat shares $\tilde{S}_p$ that party p could *plausibly* win given different configurations of districts producing *the same* national-level vote share V_p. By spreading voter support thinly across districts or concentrating it in just a few districts, it is possible for parties receiving identical national-level vote shares to win very different national-level seat shares. Which seat shares $\tilde{S}_p$ are more or less plausible at each V_p are determined by the estimated distribution $F(Y;\widehat{\boldsymbol{\pi}},\widehat{\boldsymbol{\mu}},\widehat{\boldsymbol{\Sigma}})$. Following Gelman and King (1994, 535), an estimate of the swing ratio for party p may be calculated as

$$\widehat{\beta}_p = \frac{E(\tilde{S}_p|\bar{V}_p+0.01)-E(\tilde{S}_p|\bar{V}_p-0.01)}{0.02}, \tag{4}$$

the average per-unit change in seat share around the total party vote $\bar{V}_p$. In the context of two-party competition, Gelman and King (1994) refer to this quantity as the responsiveness, although they calculate it around the average district vote $\bar{v}_p = \frac{1}{D}\sum_{i=1}^{D} v_{ip}$ rather than the total vote $\bar{V}_p$. It is implicit in equation (4) that there are a range of plausible seat swings for each party given a 1% change in their vote share. The swing ratio estimate $\widehat{\beta}_p$ represents only the most typical of these.

Monte Carlo simulation provides a means to computationally evaluate $E(\tilde{S}_p|\bar{V}_p+0.01)$ and $E(\tilde{S}_p|\bar{V}_p-0.01)$. The procedure begins by resampling hypothetical district-level election outcomes from $F(Y;\widehat{\boldsymbol{\pi}},\widehat{\boldsymbol{\mu}},\widehat{\boldsymbol{\Sigma}})$, which reflects variation in both turnout decisions and vote choice. Aggregating these simulated districts, and repeating the process a large number of times, produces a set of hypothetical vote–seat pairs $(\tilde{V}_p,\tilde{S}_p)$ that are supported by the observed data. Using estimates of $\widehat{\boldsymbol{\pi}}^m$, $\widehat{\boldsymbol{\mu}}^m$, and $\widehat{\boldsymbol{\Sigma}}^m$ from each pattern of party contestation, the simulation proceeds as follows:

1. Starting with the first pattern of party contestation, select one of the R_m component distributions by sampling once from a multinomial distribution with probabilities $\widehat{\boldsymbol{\pi}}^m$.
2. Draw $\tilde{Y}_j$ from the selected component distribution to create a hypothetical district characterized by the number of votes cast, $\tilde{z}_j$, and $P_m - 1$ log vote ratios, $\tilde{\lambda}_{j2}, \tilde{\lambda}_{j3}, \ldots, \tilde{\lambda}_{jP_m}$. In the rare case of a simulated $\tilde{z}_j<0$, $\tilde{z}_j$ can be set to zero. (Subscripts j differentiate simulated districts from observed districts indexed by i.)
3. Repeat this process until each pattern of contestation has N_m simulated districts.
4. Stack the D^c simulated districts across all patterns of contestation.
5. Transform the party log vote ratios back into units of party vote shares, such that $\tilde{v}_{jp} = e^{\tilde{\lambda}_{jp}} / \sum_{q=1}^{P_m} e^{\tilde{\lambda}_{jq}}$ within each district. Since party one was chosen as the reference party, $\tilde{\lambda}_{j1} = 0$ by definition. If a party did not compete in the mth pattern of contestation, its vote share is set to zero.
6. Multiply together the simulated district sizes and party vote shares, $\tilde{z}_j\tilde{v}_{jp}$, to find the hypothetical number of votes cast for each party in each contested district. This accounts for malapportionment.
7. Calculate parties' simulated national-level vote shares, $\tilde{V}_p$, by dividing their simulated vote total, $\sum_{j=1}^{D^c}\tilde{z}_j\tilde{v}_{jp}$, by the simulated number of votes cast for all parties, $\sum_{j=1}^{D^c}\tilde{z}_j$.
8. Calculate parties' simulated national-level seat shares, $\tilde{S}_p$, as the number of hypothetical districts in which the party received a plurality of votes, plus the number of districts (set aside from above) in which the party ran unopposed, divided by D.[7]

This entire simulation process is then repeated, each time recording the national-level vote share and seat share received by each party in each hypothetical election, $(\tilde{V}_p,\tilde{S}_p)$. Each simulated election consists of a number of random districts equal to the actual number of election districts, in proportion to the observed pattern of party contestation. By simulating elections as a whole, the method obviates the need for preliminary stage estimation of the variability in individual districts' vote outcomes. As an additional confirmation that $F(Y;\widehat{\boldsymbol{\pi}},\widehat{\boldsymbol{\mu}},\widehat{\boldsymbol{\Sigma}})$ fits the data, it is possible at this stage to create a series of Q-Q plots comparing the simulated marginal distributions of party vote shares to the observed distributions.

[7]Modifying this seat allocation rule to mimic the various methods used in proportional representation systems (e.g., d'Hondt, Sainte-Laguë, multi-tier, etc.) would make it possible to test the expectation that parties' swing ratios approximate one in those democracies.

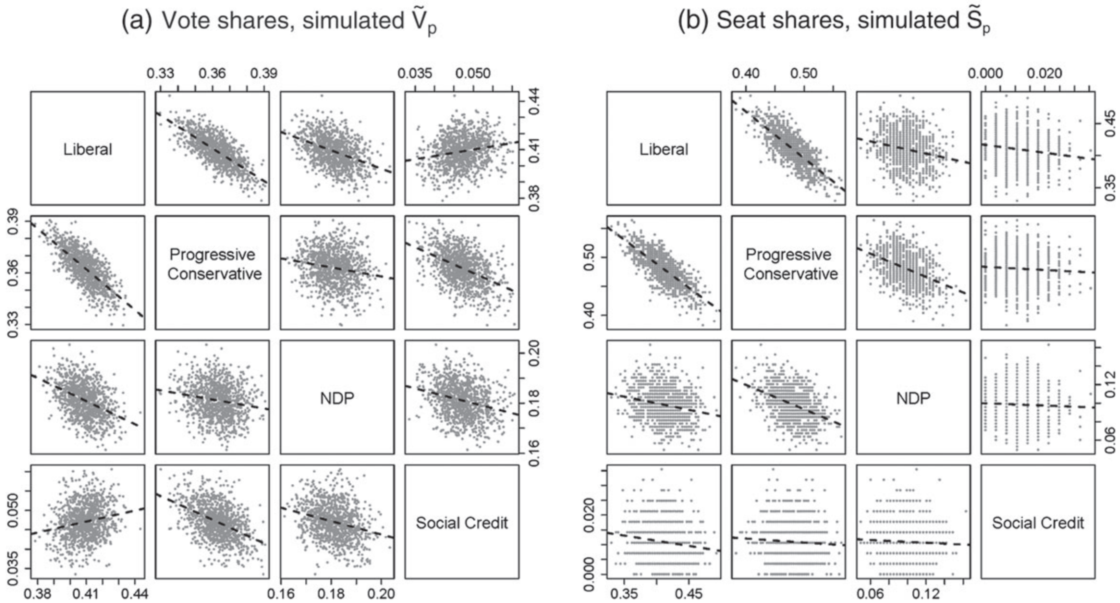

Fig. 4 Relationships among (a) hypothetical national-level vote shares $\tilde{V}_p$; and (b) seat shares $\tilde{S}_p$ for four parties in the Canadian federal election of 1979. Results show 1000 elections simulated from the model in Fig. 2. Dashed lines in each panel represent the fit of a linear regression.

Calculation of the swing ratios for each of the parties (equation 4) directly follows. To find $E(\tilde{S}_p|\bar{V}_p+0.01)$, calculate the average value of $\tilde{S}_p$ for simulated election outcomes in which $\tilde{V}_p \approx \bar{V}_p+0.01$. Similarly, $E(\tilde{S}_p|\bar{V}_p-0.01)$ is the average value of $\tilde{S}_p$ for simulations in which $\tilde{V}_p \approx \bar{V}_p-0.01$. A range of ± 0.002 around $\bar{V}_p+0.01$ and $\bar{V}_p-0.01$ will produce stable estimates of $\tilde{S}_p$ for sufficiently large numbers of simulated election outcomes. The narrow range is necessary because the simulation results will be asymmetrical around each value.

Although equation (4) requires no parametric assumptions about the functional relationship between $\tilde{V}_p$ and $E(\tilde{S}_p|\tilde{V}_p)$, the relationship between simulated seat shares $\tilde{S}_p$ and simulated vote shares $\tilde{V}_p$ around $\bar{V}_p$ is oftentimes approximately linear. In that event, the slope of a linear regression of $\tilde{S}_p$ on $\tilde{V}_p$ will be roughly equivalent to the swing ratio estimate $\widehat{\beta}_p$.

To conclude the example, in the 1979 Canadian federal election, both the Progressive Conservative and Liberal Parties could expect to gain or lose approximately 2% of the total number of seats in the House of Commons, for a 1% change in vote share (Fig. 3). The swing ratio for the New Democratic Party was slightly lower, with an expected change of about 1.8% of legislative seats for a 1% change in vote share. Because of its small size and the way in which support for the last-place Social Credit Party was distributed across districts, a 1% change in vote share only predicts a 0.7% change in seat share for that party.

3.4 *Interrelationships among Parties' Vote and Seat Shares*

A feature of the simulation process is that it reveals how parties' national-level vote and seat shares are expected to covary, based upon the distribution of their district-level support. Multiparty swing ratios predict the change in each party's legislative seat share if that party wins a different share of the popular vote. But party vote shares are bound by a sum-to-one constraint: when one party increases its share of the vote, those votes must be drawn somehow from the other competing parties. The same constraint applies to parties' seat shares. Plotting the different values of $\tilde{V}_p$ and $\tilde{S}_p$ across a large number of simulations visualizes the interrelationships among simulated national-level party vote and seat shares.

For example, in the 1979 Canadian federal election, increases in the share of the vote received by the Liberal Party would be offset—on average—by large decreases in the share of the vote going to the Progressive Conservative Party, smaller decreases in the vote share of the NDP, and *increases* in the share won by the Social Credit Party (Fig. 4). This reflects the fact that in 1979, Social Credit won all six of its

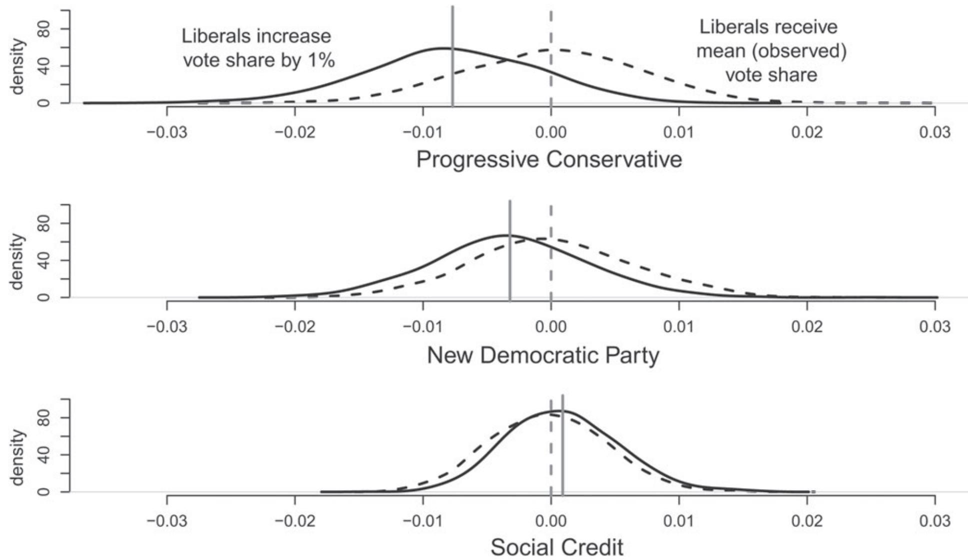

Fig. 5 Kernel density plots showing the estimated effect of a 1% increase in the Liberal Party's national-level vote share on changes in the vote shares of the other three parties. Horizontal axes have been recentered over zero, representing no change from the average simulated $\tilde{V}_p$ for each party. The dashed curves give the distribution of each party's simulated $\tilde{V}_p$ at the mean simulated vote share of the Liberal Party. The solid curves give the distribution of each party's simulated $\tilde{V}_p$ when the Liberal Party's simulated vote shares are 1% above their mean. Vertical lines denote the means of each distribution.

seats in Quebec, where the Liberals also ran extremely strong. In part because of this geographic connection, however, increases in the share of the seats won by the Liberal Party tend to come at the expense of *all three* of its competitors, including Social Credit.

Figure 5 summarizes the full range of vote swings plausibly associated with a 1% increase in the share of the national-level vote won by the Liberal Party. These are estimates of the marginal distributions $\Pr\left(V_p \middle| V_{\text{liberal}} = \bar{V}_{\text{liberal}} + 0.01\right)$. In expectation, the Progressive Conservative Party loses 0.8% of the vote but could plausibly lose as much as 2% or even gain 1%. Similar ranges are evident for the other two parties. Even if the Liberal vote share remains unchanged in a subsequent election, the distributions marked by dotted lines still indicate a variety of ways in which the vote shares of the other three parties might vary.

The 1979 Canadian federal election was rapidly followed by another election in 1980, as the result of a vote of no confidence in the minority government of the Progressive Conservative Party. Between the two elections, the Liberals increased their popular vote share by 4.4% (calculated as a share of the major party vote), whereas the Progressive Conservatives lost 3.4%. Dividing these two changes results in an actual loss of 0.8% of the vote for the Progressive Conservative Party for a 1% increase in the vote for the Liberals, exactly as predicted by the model. The NDP gained 2% of the vote from 1979 to 1980 or 0.4% per 1% increase in the Liberal vote share, well within the predicted distribution of NDP vote swings in Fig. 5. For the Social Credit Party, the change per 1% increase in the Liberal Party vote share was −0.7%, again consistent with the predictions of the model.

3.5 *The Seats–Votes Curve and Partisan Bias*

The hypothetical vote–seat pairs $(\tilde{V}_p, \tilde{S}_p)$ produced by the simulation may be further analyzed to investigate two other important features of electoral systems: the seats–votes curve and partisan bias. Although simulation results are only needed in a 2% range of vote shares around $\bar{V}_p$ to estimate each party's swing ratio, in practice, simulated values of $\tilde{V}_p$ can extend 10% or more around the actual election outcome. This permits estimation of each party's (local) seats–votes curve near $\bar{V}_p$; that is, the value of $E(\tilde{S}_p | V_p)$, as well as a confidence interval around this estimate.[8] The expected seat share at a given V_p is the mean of the simulated $\tilde{S}_p$ for $\tilde{V}_p \approx V_p$. A (say) 95% confidence interval is obtained by sorting the respective $\tilde{S}_p$ and selecting the simulated values at the 2.5 and 97.5 percentiles.

The local seats–votes curves for parties in the 1979 Canadian federal election are plotted in Fig. 3. For the two major parties, the confidence intervals extend approximately ±3.5% in either vertical direction around the seats–votes curves, corresponding to ±10 seats out of 282. As noted above, it is often the case that a linear approximation is quite close to the estimated seats–votes curve. The curves each pass through the actual national-level election result, which again verifies the model fit and validates the simulation technique.

The extent of partisan *bias* in a given election reflects the systematic advantage in seat share allocation a party receives relative to other parties by virtue of the electoral rules in place and the manner in which its supporters are distributed across districts. In the 1979 Canadian federal election, for example, the Progressive Conservative Party clearly enjoyed an advantage over the Liberal Party, winning 8% more of the legislative seats despite receiving 4% fewer of the vote.

In two-party elections, where parties tend to receive close to 50% of the vote, a standard definition of partisan bias is "the expected proportion of the seats over 0.5" received by a party "when they receive exactly half the average district vote" (Gelman and King 1994, 536). My method permits estimation of this quantity, but only if parties' actual vote shares are in the proximity of 50%—which, in multiparty legislative elections, is almost never the case. As such, the simulation will not produce estimates of $\tilde{V}_p \approx 0.5$, and predictions of $\tilde{S}_p$ at those values will be beyond the scope of the data. This definition of partisan bias is therefore of limited use in the multiparty setting.[9] More generally, although the New Democratic Party, for example, received a smaller share of the seats than its share of votes, this is to be expected for a small party in a high-responsiveness majoritarian system. Evidence of bias against the NDP would be if the Progressive Conservative or Liberal Parties received a larger share of the seats, had they won the same share of the votes as the NDP. Yet, again, the results of only one election do not provide enough information to support inference about that extreme counterfactual scenario.

I propose instead that in systems where parties have the potential to win an outright majority in the legislature, an informative alternative measure of partisan bias is the difference in *vote shares* required of the leading parties to obtain 50% of the seats. This measure will only apply to the largest parties running for office and will not apply at all in some countries. By this standard, the Progressive Conservative Party would have only needed one additional percentage of the national popular vote, or 37%, to win a majority of seats in the House of Commons. In contrast, the Liberal Party would have needed another 5% of the national vote to reach a majority in the House, or a total of almost 46%. In 1979, then, the electoral bias against the Liberal Party, due to the particular cross-district distribution of its supporters, was approximately 9%.

4 Swing Ratios in the United States, United Kingdom, and Botswana

To demonstrate the applicability of the model more broadly, I evaluate historical swing ratios for parties in three countries employing first-past-the-post electoral rules: the United States, the United Kingdom, and Botswana.[10] Analyzing elections to the U.S. House of Representatives permits a demonstration of how, although the approach is designed to apply in general to systems with any number of political parties, it can easily scale downward when only two parties contest an election. As Gelman and King (1994) also estimated electoral responsiveness in the U.S. House using their model of two-party competition, this further enables me to test the validity of my measure by comparing my estimates to theirs.

[8]Note that the method does not support extreme counterfactual predictions about party seat shares for vote shares far from the observed election outcome—nor should it, considering that it only examines results from one election at a time. In a similar manner, Gelman and King (1994, 534), as well as subsequent researchers, choose to restrict the use of their model to party votes in a relatively narrow interval around the actual election outcome.

[9]An extension of this logic to the three-party case might compare the expected share of the seats won by each party if all vote shares equaled 33%, but again, this is not close to actual multiparty election outcomes where large parties compete against small parties. In the four-party case of the 1979 Canadian federal election, only the New Democratic Party received close to 25% of the vote; the other three parties were either well above or below this amount.

[10]District-level election results for each country are obtained from the Kollman et al. (2010) *Constituency-Level Elections Archive*.

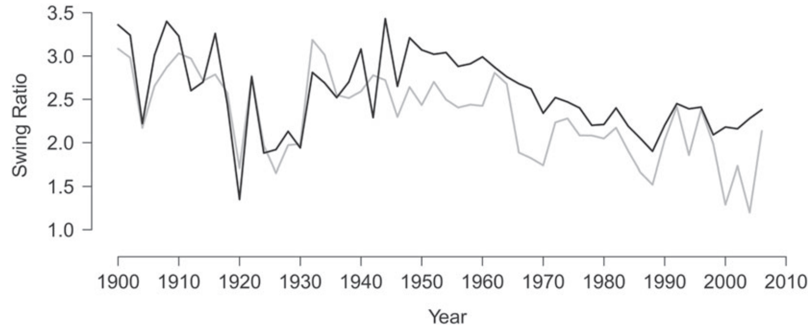

Fig. 6 Swing ratios in elections to the U.S. House of Representatives, non-southern states, 1900–2006. The black line represents estimates produced by the method described in this article; the gray line represents estimates produced by the method of Gelman and King (1994).

Parliamentary elections in the United Kingdom, unlike in the United States, are fought between two major political parties in addition to a significant third party and a range of smaller and regional parties. The party system has also been less stable from year to year than that of the United States. Finally, Botswana is a multiparty democracy in which one party dominates. Since achieving independence in 1966, the Botswana Democratic Party (BDP) has won every election with an outright majority of the vote. From a methodological standpoint, Botswana is interesting because it contains many fewer—and smaller—electoral districts, compared to the United States and United Kingdom.

4.1 *Swing Ratios in the U.S. House of Representatives*

A long-standing concern in the study of American politics has been the apparent decline in electoral responsiveness in the United States House of Representatives from the 1960s onward (Tufte 1973, 1975; Brady and Grofman 1991). Scholars agree that the incumbency advantage for members of Congress increased during that time (Alford and Hibbing 1981; Gelman and King 1990a) but have debated why this happened and what it implies for electoral competition and the potential for legislative turnover (Erikson 1972; Mayhew 1974; Ferejohn 1977; Collie 1981; Jacobson 1987; Abramowitz 1991; Ansolabehere, Brady, and Fiorina 1992; Cox and Katz 1996).

To estimate the swing ratio for the Democratic and Republican parties in elections to the U.S. House of Representatives, I apply the method described in this article to district-level election outcomes from 1900 to 2006. In doing so, I replicate and extend the analysis of Gelman and King (1994), who used a specialized two-party model to estimate the same quantity through 1992. Recall that in two-party systems, the party swing ratios are equivalent to the responsiveness in the seats–votes relationship around the observed election outcome. It is expected that the two procedures will produce similar results. Like Gelman and King, I confine my analysis to non-southern states, excluding a large number of districts that were dominated by Democrats and uncompetitive during much of the twentieth century.[11]

For each election, I approximate the joint distribution of party log vote ratios and district sizes in contested districts using a three-component mixture model. To remain consistent with Gelman and King (1994), I treat districts in which either party received fewer than 5% of the two-party vote as uncontested. I then simulate 10,000 hypothetical elections per year and use equation (4) to estimate the swing ratio. The estimates I produce are relatively stable from year to year, an indication that the method is robust despite only analyzing one election at a time (Fig. 6).

Comparing my estimates to a series produced by the method of Gelman and King, the two lines clearly trend together, despite discrepancies due to differing modeling assumptions—in particular, how the two methods handle uncontested districts, how the parties' national-level vote share is calculated (as the average district vote or as the total vote share), and different assumptions about district-level variation

[11]Following Gelman and King (1994), the ten southern states are Alabama, Arkansas, Georgia, Florida, Louisiana, Mississippi, North Carolina, South Carolina, Texas, and Virginia. Retaining these states in the analysis would generate lower swing ratio estimates.

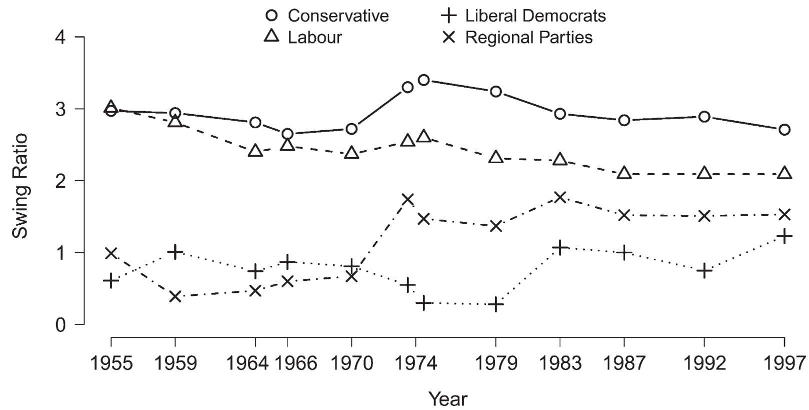

Fig. 7 Swing ratios for parties in U.K. parliamentary elections, 1955–1997. The party labeled Liberal Democrats was the Liberal Party prior to the 1983 election; in 1983 and 1987, they competed as the Social Democratic Party–Liberal Alliance.

in election outcomes.[12] Both measures confirm that the swing ratio among non-southern districts first increased from 1920 to the mid-1940s, peaking at over 3, and then enduring a gradual 40-year decline. By 1988, the swing ratio had reached its lowest point in almost 60 years. The trend since the early 1990s, however, appears to be one of increasing electoral responsiveness, perhaps foretelling greater legislative turnover in the U.S. House of Representatives.

As the swing ratio is a conceptual quantity, it is difficult to conclude that either estimate is the *correct* one, though they are close. As emphasized above, there are actually a range of plausible swing ratios that are supported by the data in each year, arising from the many different ways in which parties' district-level vote shares could change to produce a 1% change in votes at the national level. Each of these different district-level configurations will result in a unique allocation of seat shares. The reported swing ratio is a summary of these different possible outcomes.

4.2 *Swing Ratios in the U.K. Parliament*

Since 1955, parliamentary elections in the United Kingdom have been contested by two major parties (Conservative and Labour), a third party receiving as much as 25% of the vote (Liberal Democrats/Social Democratic Party–Liberal Alliance/Liberal), and a large number of minor national parties and regional parties in Northern Ireland, Scotland, and Wales. Historically, the Conservative and Labour parties, as the two largest, trade off legislative seats in a nearly one-to-one fashion: across 12 elections from 1955 to 1997, the correlation between those two parties' seat shares is -0.94. But it is only since the elections of 1974 that the two parties' vote shares have been similarly highly correlated. This is due to the changing ways in which the third party and regional parties draw votes from the other two, despite never being large enough to earn a significant share of the seats.

I assess the consequences of British electoral geography for parties' evolving seats–votes relationships. Since regional parties regularly win seats in the House of Commons, and as their supporters are distributed geographically in a manner systematically related to the three large parties, I retain those parties in the analysis but combine their district-level vote shares to facilitate estimation of the major parties' swing ratios. The results indicate a clear pre-/post-1974 break in British electoral politics (Fig. 7), consistent with the observation by Norris and Crewe (1994, 207) that "over time . . . the electoral system has become less exaggerative of the difference between the major parties in government and opposition."

Since 1974, the Conservative Party's swing ratio has exceeded that of the Labour Party by between 0.6% and 1.0% of legislative seats per percent of the vote. This pattern may be viewed as either an advantage or a disadvantage for the Conservative Party. Once elected, Conservative MPs are more

[12]I implement the Gelman and King model using the R package JudgeIt (Gelman, King, and Thomas 2008). Additional differences between the replicated gray line in Fig. 6 and the estimates reported in Gelman and King (1994) arise because I use a more complete and accurate data set of election results (Kollman et al. 2010) and do not include incumbency as a covariate, which my method does not support.

vulnerable to losing their seats as a result of small decreases in their party's overall level of popular support. But for Conservative challengers, small increases in their party's overall share of the vote are expected to produce larger parliamentary seat gains, relative to equivalent increases in Labour Party vote shares. Election results since 1974 conform to these model predictions: the variance in Conservative seat shares has been greater than the variance in Labour seat shares, despite the fact that the variance in Conservative vote shares is smaller than the variance in Liberal vote shares. As originally demonstrated by Tufte (1973, 550–552), higher swing ratios are associated with greater legislative turnover.

The difference between Conservative and Labour swing ratios has persisted despite the changing electoral fortunes of both parties. Across seven elections from 1974 to 1997, the Conservative and Labour Parties' national-level vote shares each fluctuated in a range of 30%–45%. Yet, both parties' swing ratios remained consistent, with no apparent relationship between vote share and swing ratio. In the two-party case, Gelman and King (1994, 535) noted that the swing ratio ''tracks with'' parties' vote shares. Evidence from the United Kingdom indicates that this is not necessarily true in general. Between the elections of 1970 and February 1974, for example, the Liberal Party increased its vote share from 7% to 19%, with associated decreases in the vote shares of the Labour and Conservative Parties. Despite its considerable improvement at the polls, the Liberal Party's swing ratio declined slightly. At the same time, the Conservative Party and the Ulster Unionist Party of Northern Ireland formally split. As a result, the vote share of the Conservative Party fell, but the swing ratios of the Conservative Party as well as the regional parties both increased. These results illustrate that parties' swing ratios are as much a function of how parties' supporters are distributed across districts as they are tied to parties' national-level vote shares.

4.3 *Swing Ratios in the 2004 Botswana Legislative Election*

In many democracies, a single party consistently wins reelection despite the presence of free multiparty elections. A key question when confronting dominant-party systems such as these is the extent to which the governing party is returned to office on the basis of their popular support or as the result of strategic manipulations to the electoral system that provide the dominant party a structural advantage in that country's elections. If, due to the drawing of district borders and the electoral rules in place, dominant parties' legislative seat shares are not sufficiently responsive to changes in parties' vote shares, then the electoral system is, in effect, securing the party's hold on power. Especially in the absence of a unified opposition, elected leaders would be expected to be less constrained in their policy choices as a result and the quality of democratic representation and accountability to suffer.

Botswana is one such dominant-party democracy, in which the governing BDP has won every legislative election in the country's history. In the 2004 election to the Botswana National Assembly, seven parties nominated candidates. The BDP took 52% of the popular vote and—owing to Botswana's majoritarian electoral system—fully 77% of the seats. How responsive is that seat share to possible changes in the BDP's vote share? If, as the dominant party, the BDP has a swing ratio of between two and three, then it is comparable to those of the governing parties in the United States and United Kingdom, and democratic observers may worry less about the possibility that the BDP is insulated from losing office. If, on the other hand, the BDP's swing ratio is closer to zero, then the BDP's share of the legislative seats is fairly protected, and it becomes debatable as to just how democratic Botswana actually is.

With 57 seats in the legislature, and an average in 2004 of just 7235 votes cast per electoral district, the Botswana National Assembly is an order of magnitude smaller than the national legislatures of the United States and United Kingdom. In addition to the 44 seats won by the BDP in 2004, the Botswana National Front (BNF) received 12 seats and the Botswana Congress Party (BCP) took the final one. Those three parties, plus the Botswana Alliance Movement (BAM) and Botswana People's Party (BPP), which had jointly entered into an electoral pact with the BNF, combined to receive over 99% of the national vote. Because the BAM, BPP, and BNF formed an alliance and did not field candidates against one another in any of the 57 districts, I aggregate their vote totals under the BNF label. I set aside the remaining smaller parties and independents.

Among the BDP, BNF-led coalition, and BCP, there were four distinct patterns of party contestation ranging in size from one district in which the BDP ran unopposed, to 48 districts in which candidates were fielded by all three parties. With so few districts in each pattern of party contestation, a single multivariate normal distribution—rather than a mixture distribution—is sufficient to provide the necessary density estimates for each. In the above notation, $R_m = 1$ for all patterns of party contestation.

Following the simulation procedure I have described, I estimate swing ratios of 2.4 for the BDP, 1.8 for the BNF-led coalition, and 1.2 for the BCP—in line with what was found for parties in the United States and United Kingdom. Considering that the BNF-led coalition won 32% of the popular vote and the BCP won 16%, swing ratios of this magnitude indicate a significant amount of electoral competition. Although the margin of victory for the BDP was quite large in 2004, their legislative seat share was responsive at the margin to changes in their popular vote share. What if the opposition had been united? In fact, the BCP had split from the BNF in 1998. Assuming that a BCP–BNF merger would not change the district-level vote shares for the BDP, my method indicates that the swing ratios for both the BDP- and a BNF-BCP–led coalition would be approximately 3.1. The BDP would still win a majority of the vote but only 58% of the seats. The results further indicate that the electoral system would be unbiased, with both parties requiring 50% of the votes to win 50% of the seats.

5 Conclusion

The success of national political parties depends on their ability not only to win votes but also to convert those votes, district by district, into the largest possible share of seats in the legislature. Accountability for these parties demands that when voters withdraw their support in future elections, parties' seat shares should decrease by a commensurate amount. And, when parties increase their vote shares in a future election, their seat shares should similarly reflect such gains. Electoral accountability and political opportunity are thus bound together by the nature of the relationship between parties' votes and seats.

This article introduces a general statistical procedure for estimating the relationship between political parties' popular vote shares and legislative seat shares—what is known as the seats–votes swing ratio. In multiparty systems, each party will have a unique swing ratio, which varies as a function of the country's electoral rules, the level of popular support for each party, and the distribution of parties' supporters across electoral districts. This method for estimating parties' swing ratios may be applied to systems with any number of political parties (including just two), using any electoral rule (including various forms of proportional representation), and can easily accommodate uncontested districts and multiple patterns of party contestation at the district level. Since the model requires district-level voting data from only a single legislative election, it is both feasible to estimate and capable of detecting potential changes in parties' swing ratios as the level and distribution of parties' popular support varies between elections. Being able to estimate multiparty swing ratios in this manner makes it possible to investigate in far greater depth the political consequences of electoral systems and electoral geography in democracies around the world.

Under majoritarian electoral rules, parties winning equal shares of the votes will not necessarily receive equal shares of the seats; nor will changes in parties' vote shares produce equal swings in those parties' seat shares. How sensitive are parties' legislative seat shares to potential changes in their popular vote shares? I have demonstrated that in four majoritarian democracies (Botswana, Canada, the United States, and the United Kingdom), parties typically have swing ratios from approximately one-half to over 3% of legislative seats per 1% change in the share of the vote. Differences in how parties' supporters are distributed geographically imply that not all parties contest legislative elections on equal footing—or even face the same electoral incentives once in office. The magnitude of these disparities has important consequences for the form and amount of representation afforded to different groups, interests, and regions and hence the amount of political power wielded by the parties that represent those blocs of voters.

Competitive democratic systems must ensure that all parties—but especially those in power—have a large enough swing ratio to permit alternation in party control of the government if sufficient numbers of voters switch parties between elections. If, instead, the design of electoral systems—including how electoral districts are drawn and how many representatives are elected from each district—limits turnover in office and insulates incumbent politicians from changes in voters' preferences for representation, then the mere presence of multiparty elections may be insufficient to produce government accountability, voter–legislator policy congruence, and other expected benefits of representative democracy.

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